Analysis of This Inclined Rectangular Aperture with Arbitrary Location in Rectangular Waveguide

Ron Yang and A. S. Omar, *Senior Member, IEEE*

***Abstract*—An analysis is developed for calculating the coupling parameters of a thin inclined rectangular coupling aperture arbitrarily located in the transverse plane of a rectangular waveguide. The analysis uses the aperture basis 2-function approach of the moment method. The calculated results for different aperture geometries and for a X-band band-pass filter realized with three resonant apertures have been confirmed to a good accuracy with experiments.**

I. Introduction

Waveguide aperture coupling has many applications for filters and impedance matching networks. Compared to the commonly used inductive windows, the rectangular coupling aperture provides the advantage of a larger susceptance range which extends from inductive to capacitive values. Besides, the resonance characteristics of the rectangular aperture can be employed to build compact waveguide band-pass filters with wide stopband.

Rectangular coupling apertures in rectangular waveguide have been extensively studied by different authors with different methods [1]–[6]. Most of the previously published works however, have been limited to the case of a rectangular aperture with its sides in parallel to the waveguide walls and mostly without offset.

In this paper, we present an analysis for an inclined rectangular aperture with arbitrary location in the transverse plane of a rectangular waveguide by using the aperture basis function approach of the moment method. The calculated susceptance values for different apertures compare very well with the experiment. An X-band band-pass filter with three resonance rectangular apertures has been simulated. Good agreement is found between the simulated filter response and the experiment.

II. Formulation

The structure under consideration is shown in Fig. 1. The metal iris forming the coupling aperture is located in the waveguide transverse plane at $z = 0$, and is assumed to be infinitely thin and perfectly conducting. The dimensions of the waveguide are such that only the dominant TE_{10}-mode can propagate unattenuated in the waveguide. The size of the aperture is $a_0 \times b_0$, where neither a_0 nor b_0 is necessarily smaller than the operating wavelength. Furthermore, α is the inclination angle of the aperture in the waveguide coordinate system X–Y. By rotating the coordinate system X–Y clockwise with angle α, a coordinate system $X' - Y'$ is created in which the sides of the coupling aperture are in parallel to the system axis, where (x_0, y_0) and (x_0', y_0') denote, respectively, the coordinates of the lower left corner of the coupling aperture in their respective systems. The coordinate transforming relations between the two coordinate systems read:

$$x' = x\cos\alpha + y\sin\alpha, \qquad y' = -x\sin\alpha + y\cos\alpha,$$

and

$$x = x'\cos\alpha - y'\sin\alpha, \qquad y = x'\cos\alpha + y'\sin\alpha\,.$$

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The authors are with the Arbeitsbereich Hochfrequenztechnik, Technische Universität Hamburg–Harburg, 2100 Hamburg 90, Germany.

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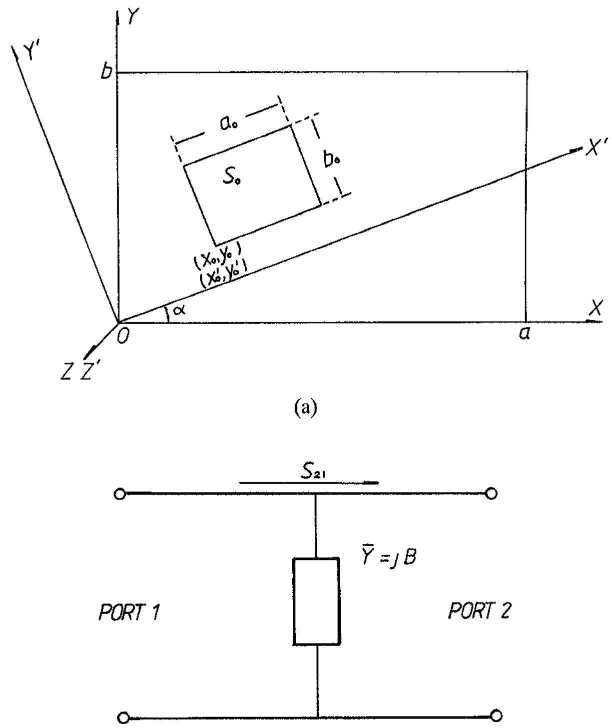

Fig. 1. (a) Inclined rectangular aperture in the transverse plane of a rectangular waveguide. (b) Equivalent Circuit.

The transverse waveguide electric and magnetic fields at both sides of the coupling aperture can be expressed as the expansions of their eigenmodes

$$\vec{E}_j = \sum_n (a_{jn} + b_{jn})\vec{E}_n, \qquad \vec{H}_j = \sum_n \pm (a_{jn} - b_{jn})\vec{H}_n$$

where $j = 1, 2$ corresponds to the waveguides at either side of the coupling aperture and

$$\int_s \vec{E}_m \cdot \vec{E}_n \, ds = W_{mn}\delta_{mn}, \qquad \int_s \vec{E}_m \cdot \vec{H}_n \, ds = \delta_{mn}$$

where s is the cross-section of the waveguide, and δ_{mn} is the Kronecker delta function.

The tangential electrical field E_{ap} in the coupling aperture is expressed as the sum of a set of basis functions which take the edge singular properties into consideration

$$\vec{E}_{ap} = \sum_\ell \sum_m d_{lm}(e_{xlm}\vec{x} + e_{ylm}\vec{y})$$

According [7] this set of basis functions in the coordinate system X'–Y' can take the form

$$e_{xlm} = \frac{\cos[\ell\pi(x' - x_0')/a_0]}{\sqrt{(x' - x_0')[1 - (x'x_0')/a_0]}} \cdot \frac{\sin[m\pi(y' - y_0')/b_0]}{\sqrt{(y' - y_0')[1 - (y' - y_0')/b_0]}}$$

$$e_{ylm} = \frac{\sin[\ell\pi(x' - x_0')/a_0]}{\sqrt{(x' - x_0')[1 - (x' - x_0')/a_0]}} \cdot \frac{\cos[m\pi(y' - y_0')/b_0]}{\sqrt{(y' - y_0')[1 - (y' - y_0')/b_0]}}$$

where $\ell, m = 1, 2, 3, 4 \cdots$ and $(\ell, m) \neq (0, 0)$.

In the coupling aperture plane, one matches the waveguide tangential electric field to that of the aperture field and the waveguide tangential magnetic fields at both sides to each other:

$$\vec{E}_j = \vec{E}_{ap} \quad \text{and} \quad \vec{H}_1 = \vec{H}_2 .$$

Then the following equations relating the incident and reflected modal amplitudes are obtained:

$$\begin{aligned} (\underline{a}_1 + \underline{b}_1) &= [W]^{-1}[C]\underline{d} \\ (\underline{a}_2 + \underline{b}_2) &= [W]^{-1}[C]\underline{d} \\ [K](\underline{a}_1 + \underline{b}_1) &= [K](\underline{b}_2 - \underline{a}_2) \end{aligned}$$

where matrix $[W]$ is a diagonal matrix whose elements are W_{nn} and the matrices $[C]$ and $[K]$ are defined as

$$\begin{aligned} C_{nk} &= \int_{s_0} \vec{E}_n' \cdot \vec{e}_k' \, ds , \\ K_{kn} &= \int_{s_0} \vec{e}_k{}' \cdot \left(\vec{H}_n' \times \vec{z}\right) ds \end{aligned}$$

and

$$\underline{a}_j = [a_{jn}], \qquad \underline{b}_j = [b_{jn}], \quad \text{and} \quad \underline{d} = [d_k]$$

where $\vec{z}$ is the unit vector in the propagation direction and s_0 is the area of the coupling aperture. The field components of $\vec{E}_n'$ and $\vec{H}_n'$ can be derived from waveguide eigenmodes according to the transformation relations

$$\begin{aligned} F_x' &= F_x \cos\alpha + F_y \sin\alpha , \\ F_y' &= -F_x \sin\alpha + F_y \cos\alpha . \end{aligned}$$

From (1), (2), and (3) the modal scattering matrices for the inclined iris in the transverse plane of a waveguide can be derived as

$$\begin{aligned} [S_{11}] &= [C]([C][K])^{-1}[K] - [U] , \\ [S_{21}] &= [C]([C][K])^{-1}[K] , \\ [S_{12}] &= [S_{21}] , \qquad [S_{22}] = [S_{11}] . \end{aligned}$$

where $[U]$ is a identity matrix.

III. Results

To verify the theoretical analysis measurements were carried out in an X-band waveguide for a variety of coupling geometries. The irises were made from copper sheet of thickness of 0.1 mm by photo etching. HP8501 network analyzers were employed to measure the S-parameters of the coupling structure. The normalized shunt susceptance of the coupling aperture is calculated from both theoretical and experimental S_{21} by the relation [7]

$$\overline{B} = -\frac{2\sin\Phi}{|S_{21}|}$$

where Φ is the phase angle of S_{21}.

A. Centered Aperture Without Inclination

Two cases for the centered rectangular aperture without inclination have been investigated. Fig. 2 shows the results for the two cases where the solid lines represent the calculated normalized susceptance accompanied by the measurements $(\cdots)$. It can be seen that the theoretical analysis agrees very well with the measured data.

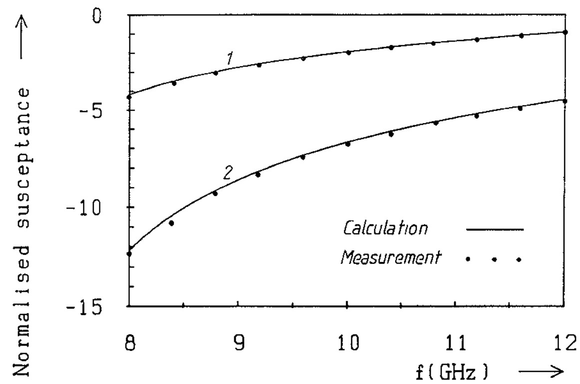

Fig. 2. Normalized susceptance for centered apertures without inclination. (1) $a_0 = 11.7$ mm, $b_0 = 3.8$ mm. (2) $a_0 = 7.8$ mm, $b_0 = 4.9$ mm.

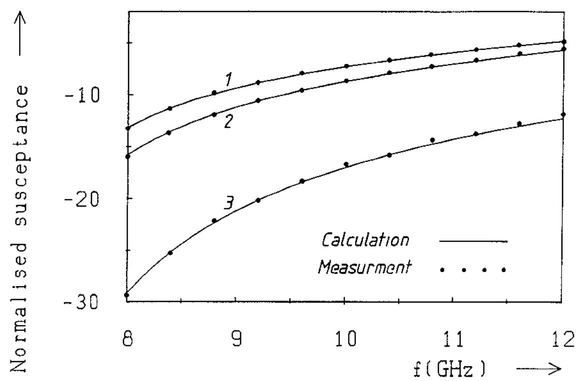

Fig. 3. Normalized susceptance for inclined apertures (1) centered, $a_0 = 7.8$ mm, $b_0 = 4.9$ mm, $\alpha = 10°$. (2) offset, center at (9.5, 5.2) in X–Y, $a_0 = 7.8$ mm, $b_0 = 4.9$ mm, $\alpha = 15°$. (3) centered, $a_0 = b_0 = 5.6$ mm, $\alpha = 45°$.

B. Inclined Aperture

For the inclined aperture three cases have been studied, among which are two cases with aperture centers coinciding with the waveguide axis and another one with an offset. Both theoretical results and measurements are shown in Fig. 3. Good agreement can also be observed.

Fig. 4 shows the normalized susceptance for three apertures of the same dimension but with different inclination angle or offset. As can be seen, with the increase of the inclination angle, an aperture tends to give a higher susceptance value. Offset aperture exhibits similar characteristics. This can be explained as follows. Inclination and offsetting of coupling apertures destroy the transverse symmetry of the rectangular waveguide. Hence, more higher order modes are excited at the vicinity of the coupling iris in order to satisfy the boundary conditions there. So more reactive, predominantly magnetic energy is stored in the region near the coupling aperture which results in a higher susceptance level.

C. Resonant Aperture

For a very narrow (very small b_0) aperture resonance can occur in the operating frequency band by adjusting the length of the aperture. Resonance characteristics can be employed to develop waveguide bandpass filters with relatively broad bandwidth by arraying such apertures quarter wavelength waveguide apart [4]. To build such filters the resonance frequency of the single aperture has to be precisely determined. Table I shows the calculated and measured resonance frequency for centered apertures of different aperture geometries.

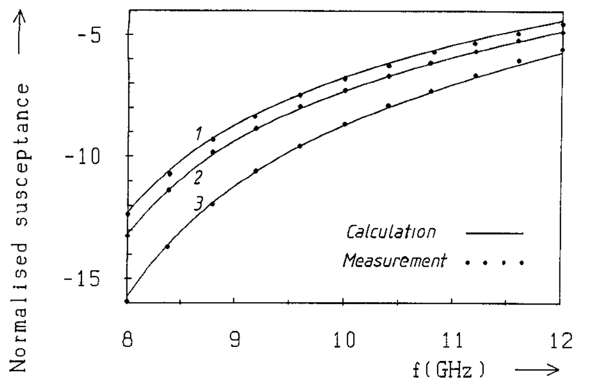

Fig. 4. Normalized susceptance for apertures with same dimension $a_0 = 7.8$ mm, $b_0 = 4.9$ mm but have different inclination and offset (1) centered: $\alpha = 0°$. (2) centered: $\alpha = 10°$. (3) offset: center at (9.5, 2.5) in X–Y, $\alpha = 15°$.

TABLE I
RESONANCE FREQUENCY FOR DIFFERENT APERTURE DIMENSIONS $\alpha = 0°$

dimension (mm)	16.9×0.9	14.8×0.5	12.9×0.9
calculation (GHz)	8.87	10.22	11.62
measurement (GHz)	8.84	10.20	11.65

Excellent agreement between theoretical and experimental results can be found.

IV. CONCLUSION

With the use of the basis function approach of the moment method the analysis for an inclined rectangular coupling aperture with arbitrary location in the transverse plane of a rectangular waveguide is carried out. It is found that an inclination or an offset of a rectangular coupling aperture leads to a higher susceptance value. With an accurate determination of the resonance frequency of the aperture, compact bandpass filters with relatively broad bandwidth can be realized. The numerical results for both a single aperture and the filter agree very well with the experiment.

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Electric Fields of an H-Plane Tapered Iris

John R. Natzke and T. Koryu Ishii

***Abstract*— Microwave electric fields of an X-band H-plane tapered iris are calculated and plotted using the moment method for the first time. The moment method results are compared with previously obtained experimental measurements and numerical results based on an equivalent circuit approach, giving confirmation that the tapered iris is both a reciprocal and asymmetrical network. The moment method results now reveal that the asymmetry stems from the asymmetry in the phase of the input and output voltage reflection coefficients, their magnitudes being equal.**

I. INTRODUCTION

Since the H-plane tapered iris exhibits reciprocal but asymmetrical transmission within a certain frequency range, it has attracted technological curiosity [1]–[3]. The past work has either been experimental [1], [2] or based on an equivalent circuit approach [3] and has not considered the electromagnetic field behavior in the proximity of the tapered iris. Until this is actually done, the reciprocal and asymmetrical transmission characteristics cannot be fully understood.

This paper presents the calculation of the electromagnetic fields of an H-plane tapered iris by the moment method using the Green's function derived for rectangular waveguide [4]. Given the total electric field distribution in the waveguide, the scattering matrix (S-matrix) is determined. The moment method results are compared with the previously published results [2], [3] by obtaining the input and output impedance and insertion loss from the S-matrix elements of the tapered iris. Thus a theoretical explanation of the reciprocal but asymmetrical transmission characteristics of the H-plane tapered iris is introduced in this paper.

II. CALCULATION OF ELECTRIC FIELDS

Consider the H-plane tapered iris structure defined by its length L and aperture width d as shown in Fig. 1. The rectangular waveguide of width a and height b is assumed to be in TE_{10} dominant mode operation, which gives an incident field component

$$E_y^i = \sin\frac{\pi x}{a} e^{-jk_z z} \tag{14}$$

where the propagation constant $k_z = \sqrt{k^2 - (\pi/a)^2}$ and the wavenumber $k = \omega\sqrt{\mu\varepsilon}$. A time dependence of $e^{j\omega t}$ is assumed and suppressed. Taking the surface current induced on the surface S_d' of the (infinitesimally thin) diaphragms to be the source of the scattered field E_y^s, the total electric field in the waveguide is $E_y = E_y^i + E_y^s$. On applying the moment method, the Green's function for rectangular waveguide is used with N impulse basis functions over S_d' to obtain the scattered field expression [5], [6]

$$E_y^s(x,z) = -\frac{\omega\mu}{a}\sum_{i=1}^{N}\left\{ I_i \sum_{m=1}^{\infty}\frac{1}{k_{zm}}\sin\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{m\pi}{a}x_i\right) e^{-jk_{zm}|z-z_i|}\right\} \tag{15}$$

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J. R. Natzke is with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109-2122.
T. K. Ishii is with the Department of Electrical and Computer Engineering, Marquette University, Milwaukee, WI 53233.
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